

DUAL SERIES EQUATIONS WITH LAGUERRE POLYNOMIALS

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ABSTRACT: Laguerre polynomials play an important role in applied mathematics and mathematical physics; this work uses them to analyze and solve dual series equations. Two linked infinite series stated in terms of Laguerre polynomials make up dual series equations, which are prevalent in boundary value problems across semi-infinite intervals. The focus of the research is on finding the unknown coefficients and optimizing the structure of these equations by using the orthogonality and completeness properties of Laguerre polynomials. Orthogonal expansions, operational procedures, and integral transforms are some of the suitable mathematical tools that will be used to clarify complicated dual series problems. This approach improves our knowledge of special functions and how to use them to solve technical and physical problems involving exponential and radial behavior, which in turn makes it easier to get good analytical answers.

Keywords: *Dual series equations, Laguerre polynomials, Orthogonal polynomials.*

1. INTRODUCTION

The dual series equations listed below are taken into consideration:

$$\sum_{n=0}^{\infty} \frac{A_n L_n^{(\alpha)}[(x+b)^h]}{\Gamma(\alpha+n+1)} = f(x), \quad 0 < x < a, \quad (1.1)$$

$$\sum_{n=0}^{\infty} \frac{A_n L_n^{(\sigma)}[(x+b)^h]}{\Gamma(\alpha+n+\beta)} = g(x), \quad a < x < \infty, \quad (1.2)$$

In this scenario, m is a positive integer, $0 < h < \infty$, $0 \leq b < \infty$, h and b are finite constants, and $\alpha + \beta + 1 > \beta > 1 - m$, $\sigma + 1 > \alpha + \beta > 0$. A Laguerre polynomial is represented by $L_n(\alpha) [(x + b)h]$, where A_n is an unknown number and $f(x)$ and $g(x)$ are specified functions.

The following dual series equations have been solved by Srivastava:

$$\sum_{n=0}^{\infty} \frac{A_n L_n^{(\alpha)}(x)}{\Gamma(\alpha + n + 1)} = f(x), \quad 0 < x < a, \quad (1.3)$$

$$\sum_{n=0}^{\infty} \frac{A_n L_n^{(\sigma)}(x)}{\Gamma(\alpha + n + \beta)} = g(x), \quad a < x < \infty. \quad (1.4)$$

Dual series equations have a particular illustration in triple series equations. They come into being when

$$h = 1, b = 0. \quad (1.5)$$

Recent work by Lowndes and Srivastava has solved the Laguerre polynomial triple series equations. The authors offer resources for solving Laguerre polynomial-based dual and triple series equations in. For the dual series equations that are relevant to this research, Sneddon provides solutions and references.

The closed-form solution makes the dual series equations, which are novel in the literature, significant. This note's findings are demonstrated to be consistent with Srivastava's. The analysis is purely formal, and the order of integrations and summation has been changed without any explanation.

2. PRILIMINARY RESULTS

A few discoveries that are beneficial in the resolution of dual series equations will be discussed in this section. The orthogonality relation for Laguerre polynomials is given by equation and equation, which allows us to derive

$$\int_0^{\infty} x^{\alpha} e^{-x} L_n^{(\alpha)}(x) L_m^{(\alpha)}(x) dx = \frac{\Gamma(\alpha + n + 1)}{\Gamma(n + 1)} \delta_{nm}, \quad \alpha > -1, \quad (2.1)$$

where δ_{nm} is the Kronecker delta.

Equations (5) and (20) for integrals allow us to quickly determine that

$$\int_0^{\xi} x^{\alpha} (\xi - x)^{\beta + m - 2} L_n^{(\alpha)}(x) dx = \frac{\Gamma(\alpha + n + 1) \Gamma(\beta + m - 1)}{\Gamma(\alpha + \beta + m + n)} \xi^{\alpha + \beta + m - 1} L_n^{(\alpha + \beta + m - 1)}(\xi), \quad \alpha > -1, \beta + m > 1, \quad (2.2)$$

$$\int_{\xi}^{\infty} e^{-x} (x - \xi)^{\sigma - \alpha - \beta} L_n^{(\sigma)}(x) dx = \Gamma(\sigma - \alpha - \beta + 1) e^{-\xi} L_n^{\alpha + \beta - 1}(\xi), \quad \sigma + 1 > \alpha + \beta > 0. \quad (2.3)$$

Using the equation, we can say that

$$\frac{d^m}{dx^m} [x^{\alpha+m} L_n^{(\alpha+m)}(x)] = \frac{\Gamma(\alpha+m+n+1)}{\Gamma(\alpha+n+1)} x^\alpha L_n^\alpha(x). \quad (2.4)$$

3. SOLUTION OF DUAL SERIES EQUATIONS

We assume that

$$\begin{aligned} x+b &= X^{1/h}, & f(X^{1/h} - b) &= f_1(X), \\ g(X^{1/h} - b) &= g_1(X), & b^h &= c, & (a+b)^h &= d, \end{aligned} \quad (3.1)$$

Therefore, the dual series equations (1.1) and (1.2) can be expressed in the following manner:

$$\sum_{n=0}^{\infty} \frac{A_n L_n^{(\alpha)}(X)}{\Gamma(\alpha+n+1)} = f_1(X), \quad c < X < d, \quad (3.2)$$

$$\sum_{n=0}^{\infty} \frac{A_n L_n^{(\sigma)}(X)}{\Gamma(\alpha+\beta+n)} = g_1(X), \quad d < X < \infty. \quad (3.3)$$

We assume that

$$\sum_{n=0}^{\infty} \frac{A_n L_n^{(\alpha)}(X)}{\Gamma(\alpha+n+1)} = f_2(X), \quad 0 < X < c. \quad (3.4)$$

The dual series equations (3.2) and (3.3) can be expressed in the following manner by combining the series equations (3.2) and (3.4):

$$\sum_{n=0}^{\infty} \frac{A_n L_n^{(\alpha)}(X)}{\Gamma(\alpha+n+1)} = F(X), \quad 0 < X < d, \quad (3.5)$$

$$\sum_{n=0}^{\infty} \frac{A_n L_n^{(\sigma)}(X)}{\Gamma(\alpha+\beta+n)} = g_1(X), \quad d < X < \infty, \quad (3.6)$$

Where

$$F(X) = \begin{cases} f_2(X), & 0 < X < c, \\ f_1(X), & c < X < d. \end{cases} \quad (3.7)$$

Multiplying (3.5) by $X^\alpha(\xi - X)^{\beta+m-2}$, The integer m must be positive. We can get the result by applying equation (2.2) and integrating with respect to X over the interval (0, ξ), switching the order of integration.

$$\sum_{n=0}^{\infty} \frac{A_n L_n^{(\alpha+\beta+m-1)}(\xi)}{\Gamma(\alpha+\beta+m+n)} = \frac{\xi^{-\alpha-\beta-m+1}}{\Gamma(\beta+m-1)} \int_0^\xi X^\alpha(\xi - X)^{\beta+m-2} F(X) dX, \quad 0 < \xi < d, \quad (3.8)$$

Where

$$\alpha > -1, \beta + m > 1. \quad (3.9)$$

If we now multiply (3.8) by $\xi^{\alpha+\beta+m-1}$, by employing formula (2.4) and differentiating both sides m times with respect to ξ , we can ascertain that

$$\sum_{n=0}^{\infty} \frac{A_n L_n^{(\alpha+\beta-1)}(\xi)}{\Gamma(\alpha+\beta+n)} = \frac{\xi^{-\alpha-\beta+1}}{\Gamma(\beta+m-1)} \frac{d^m}{d\xi^m} \int_0^\xi X^\alpha (\xi-X)^{\beta+m-2} F(X) dX, \quad 0 < \xi < d, \quad (3.10)$$

Where

$$\alpha > -1, \beta + m > 1. \quad (3.11)$$

Multiplying (3.6) by $e^{-X}(X - \xi)^{\sigma-\alpha-\beta}$, integrating with respect to x over (ξ, ∞) and flipping the order of integrations, we can ascertain using formula (2.3).

$$\sum_{n=0}^{\infty} \frac{A_n L_n^{(\alpha+\beta-1)}(\xi)}{\Gamma(\alpha+\beta+n)} = \frac{e^\xi}{\Gamma(\sigma-\alpha-\beta+1)} \int_\xi^\infty e^{-X} (X-\xi)^{\sigma-\alpha-\beta} g_1(X) dX, \quad d < \xi < \infty, \quad (3.12)$$

where

$$\sigma + 1 > \alpha + \beta > 0. \quad (3.13)$$

The left-hand sides of (3.10) and (3.12) are now identical. Making use of the orthogonality relation (2.1), we find from (3.10) and (3.12) that

$$A_n = \Gamma(n+1) \left[\int_0^d \frac{e^{-\xi} L_n^{(\alpha+\beta-1)}(\xi) F_1(\xi) d\xi}{\Gamma(\beta+m-1)} + \int_d^\infty \frac{\xi^{\alpha+\beta-1} L_n^{(\alpha+\beta-1)}(\xi) G(\xi) d\xi}{\Gamma(\sigma-\alpha-\beta+1)} \right], \quad (3.14)$$

Where

$$F_1(\xi) = \frac{d^m}{d\xi^m} \int_0^\xi X^\alpha (\xi-X)^{\beta+m-2} F(X) dX, \quad (3.15)$$

$$G(X) = \int_\xi^\infty e^{-X} (X-\xi)^{\sigma-\alpha-\beta} g_1(X) dX, \quad (3.16)$$

Provided that $\alpha+\beta + 1 > 1 - m$ and $\sigma + 1 > \alpha+\beta > 0$.

(3.15) can be expressed in the following manner with the assistance of (3.7):

$$F_1(\xi) = \frac{d^m}{d\xi^m} \left[\int_0^c X^\alpha (\xi-X)^{\beta+m-2} f_2(X) dX + \int_X^\xi X^\alpha (\xi-X)^{\beta+m-2} f_1(X) dX \right], \quad c < \xi. \quad (3.17)$$

When we put

$$b = 0, h = 1, f_2(X) = 0 \quad (3.18)$$

The solutions to the dual series equations (1.3) and (1.4) are obtained after the solutions to the dual series equations (1.1) and (1.2) have been solved. This finding is in perfect agreement with previous research.

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